MTH 520/622: Introduction to hyperbolic geometry

Practice Assignment I

1. Try to answer all questions (marked in red) in the solutions to the midterm.
2. Let C be a circle in $\mathbb{H}$ that has center at i and radius ρ.
   (a) Where does this circle meet the imaginary axis?
   (b) Is it symmetric with respect to the imaginary axis?
   (c) What are its Euclidean center and radius?
   (d) Describe the horocycle associated with C.
3. Consider the Cayley transformation $C : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ defined by
$$z \overset{C}{\mapsto} \frac{z-i}{z+i}.$$
   (a) Show that the Cayley transformation maps $\mathbb{H}$ to $\mathbb{D}$.
   (b) Using C, derive the metric and the hyperbolic distance in $\mathbb{D}$.
4. Prove that concentric circles in $\mathbb{D}$ are exponentially close to each other in the Euclidean metric.
5. Show that $\mathrm{Isom}^+(\mathbb{H})$ acts transitively on the set of ideal triangles of $\mathbb{H}$. Is this action uniquely transitive?